

Volume 32

Journal of the Numismatic Association of Australia



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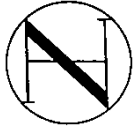
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Front cover: Photo of Mr. Billing's Gold Medal for Law (see article Figure 2 page 88).

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NUMISMATIC ASSOCIATION OF AUSTRALIA INC

President's Report

You are looking at Volume 32 of the *Journal of the Numismatic Association of Australia* (JNAA). It is a result of authors who have been sufficiently excited about a numismatic topic to carry out original research and put 'pen to paper', reviewers who have been willing to offer constructive criticisms to make the manuscripts the best they can be and the efforts of the editor who shepherds all the articles through the whole process and adds a 'final polish'. I acknowledge everyone involved, with special thanks to Managing Editor Gil Davis and Production Editor Barrie Newman for their continued efforts at maintaining a high-quality journal.

Much has happened since the last president's report published in Volume 31. The opening up of COVID-19 restrictions saw a welcome return to the NAA conference (NAAC2023), which was held in Adelaide at the Naval, Military & Air Force Club on the weekend of 19 – 20th October 2023, and hosted by the Numismatic Society of South Australia (NSSA). The conference was preceded by the 1000th meeting of the NSSA on the evening of Friday 18th October 2023. I would like to congratulate the NSSA for reaching this impressive milestone and for their major efforts in hosting the 9th NAA conference. The conference was an outstanding success, with attendees from every state of Australia and New Zealand. The conference program consisted of an opening talk by Ms Lainie Anderson (author of the *Long Flight Home*), two plenary lectures, 12 regular talks and a short talk. All talks were of a high standard and highlighted the diverse interests of the Australian and New Zealand numismatic community.

One of the highlights of the conference was the dinner, during which the Ray Jewell Silver Medal was awarded to the JNAA Managing Editor, Associate Professor Gillan Davis. This important award for 'outstanding contribution to Australian numismatics and the Numismatic Association of Australia' recognises Gil for his services to the NAA, and his numismatic research in Australia and overseas for which he is internationally renowned. Given that Gil is only the 8th recipient since the award was first presented in 1998, I have asked Walter Bloom to prepare a separate short report based on his presentation speech, which can be found in this volume.

The AGM, held at the conference, saw a ‘changing of the guard’, with Walter Bloom and Lyn Bloom handing over the reins as president and treasurer respectively. I personally want to thank Walter and Lyn for their tireless efforts in maintaining the local and international profile of the NAA over many years, including during the difficult COVID-19 period.

The following Office Bearers were elected at the 2023 AGM:

Treasurer – Rachel Mansfield

Secretary – Bridget McClean

President – Richard O’Hair

Vice President – Walter Bloom

Managing Editor – Gil Davis

I would like to thank our sponsors for their continued support of the NAA: Noble Numismatics (Gold), Coinworks, Downies (Silver), Drake Sterling, Mowbray Collectables and Sterling & Currency.

Finally, a valuable part of NAAC2023 was a round table discussion on the future of the Numismatic Association of Australia. Prior to the conference, clubs and societies were asked to send their feedback on the following:

- Any views of the Society/Club about the NAA, especially its activities and how these serve Australian numismatics.
- What would the Society/Club like to see from the NAA in the future?

The feedback received and the discussions had at the conference were valuable and the Council will work through the issues raised. The most important issue which was identified is one with which many societies are grappling: how to maintain an active membership that is willing and able to volunteer to help out with the various activities required to maintain the society. So, I would encourage all of you to think about how you might contribute to your local club or society and the NAA. We welcome your submissions to the JNAA and hope to see you at the NAAC2025 (details to appear in 2024)!

Professor Richard A. J. O’Hair

President, NAA

27 November 2023

Report on the Silver Ray Jewell Award to Associate Professor Gillan Davis

We acknowledge the important work that Associate Professor Gillan Davis, as Managing Editor of the *Journal of the Numismatic Association of Australia (JNAA)*, has undertaken for the NAA over the past 11 years. Gil has been Managing Editor of each of the Journals consecutively since 2011 – Volume 22 through to Volume 31 in 2023, and has almost completed finalising the articles for JNAA32, 2023. His contribution for all these Journals has been outstanding.

The Journal is now recognised internationally, and Gil has been instrumental in sourcing contributors and assessing their input. His attention to detail and editorial work is unsurpassed and we believe Gil should be recognised by the NAA for his support and services to the Association by being awarded the silver Ray Jewell Award.

Gil has included an Editor's Letter or Note in all the NAA Journals issued since the 2012 edition (No 23) in which he has highlighted the Journal as the showcase of the NAA, the peak body for numismatics in Australia.

He strongly promotes the NAA through each Journal and through his involvement with Macquarie University and its Australian Centre for Ancient Numismatic Studies (ACANS) and students.

Gil has sourced many of the unique articles from highly qualified Australian and international numismatic authors and supported Australian PhD students in their numismatic research in digs in Israel and the Middle East. He has ensured that there is a good mix of modern and ancients coverage throughout each Journal.

Gil was instrumental in introducing the Journal electronically in 2015 and it is now readily available to members and the public alike on our website. Through his efforts the Journal has now become a major teaching aid in subjects such as history and humanities, as he has highlighted in Journal No 30, 'teaching with numismatics – coins are useful teaching tools'.

Gil has truly supported the NAA in all his endeavours and is most deserving of the Ray Jewell silver award.

Gillan (Gil) Davis has given me (in my previous role as President) excellent advice on many issues arising in the NAA outside of his editorial expertise. I always value his input, and indeed continue to do so.

After Gil moved to the Australian Catholic University, where he is the Director of the Ancient Israel Program at the Australian Catholic University which offers a full major and minor in Archaeology to students in Arts, Education, Theology and the Ramsay Centre for Western Civilisation together with an annual dig in Israel and school outreach, he faced establishing a new degree programme, supervising students and liaising with ACANS, continuing with his Middle Eastern digs, and being an important member of the European Research Council (ERC) Advanced Grant titled ‘Silver Isotopes and the Rise of Money’, based in Lyon, which is geolocating and isotopically identifying ancient silver ore sources and matching them with coins and silver artefacts.

Gil is a personal friend and we are in frequent contact, but still I continue to be impressed about how he has managed to fit in all of these activities after a change mid-life from Real Estate to academia.



Figure 1: Presentation of the silver Ray Jewell Award by Professor Walter Bloom to Associate Professor Gillan Davis at the dinner of the NAA Conference 2023 in Adelaide, 19th August 2023.

Professor Walter Bloom
12 December 2023

Editor's note

This is an eclectic volume covering a wide range of interesting topics. The concentration is on modern material while the 'ancients' deal with Roman coins and medals. Many of the papers were also given as presentations at the highly successful Numismatic Association of Australia conference held in Adelaide earlier in the year. As always, it is a pleasure to see domestic scholarship supplemented by overseas contributors from the United States, the UK, Italy and New Zealand.

Paul Holland gives us interesting information about that perennial Australian favourite – the 1930 penny, providing details of the mint records and earliest numismatic literature. Mint records are used in another way by Eric Frazer in his analysis of the patterns of coin circulation in Australia over the last two decades. He quantifies the decline in the number of coins in circulation per person speculating on the probable phasing out of 5 and 10 cents coins. Eric provides a second and complementary article analysing the circulation of foreign coins among Australian decimal coinage and their sources of origin; the list may surprise you.

A topic that intrigues ancient through to early modern numismatists is estimating the production rate of mints. This is essential for quantification studies and it is fair to state that opinions are greatly divided. So, it is with interest that we present a detailed study by Pierluigi Debernardi on the production of denarii of Crepusius, an otherwise unknown moneyer in the Roman Republic dated to 82 BCE. The software that he has developed provides a mintage model which successfully matches the coin evidence.

While in the Roman period, we have an article by Bruce Marshall on the so-called 'Restoration' coins of the CE first century emperors Vespasian and Titus which, he argues, by reviving Augustan types, served as propaganda to justify their seizure of power. Andrew Chugg takes us into the second century with a short note updating his earlier article (*JNAA* 31) on the authenticity of some of the specimens of the medallion struck by the emperor Hadrian to commemorate his lover and favourite, the youth Antinous, who drowned in the Nile in CE 130.

Vaughn Humberstone usefully provides a comprehensive and fully referenced listing of the 45 New Zealand trade tokens issued between 1857 and 1875 together with background on the circumstances which led to them being struck despite never being legal tender and interesting details on the merchants and the dies they employed. Across the ditch and almost exactly contemporaneously, NAA President Richard O'Hair has contributed a study of the gold medal for law awarded by Mr Billing at the University of Melbourne. The research was prompted by the discovery of one of the 15 medals awarded between 1858 and 1874.

Finally we have a pair of articles dealing with remembrance by two stalwarts of the NAA. Barrie Newman, our Production Editor, shares a lovely story of his proposal to sell commemorative ingots to the United Arab Emirates for his company, The Adelaide Mint. Sadly, it did not end well. Channelling Marcel Proust à la recherche du temps perdu, Walter Bloom, President of the NAA for many years, narrates the story of his numismatic life. It is like walking through a wonderful antiquarian bookshop with a friend. Along the way, he tells the story of numismatics and coin dealers and medallists in this country, as well as his personal, often quirky, research and collecting interests.

As always, I sincerely thank the many anonymous reviewers who have reviewed the papers with special thanks to Barrie Newman for his careful attention to the role of Production Editor and John Melville-Jones for proofreading many of the articles.

On a personal note, I thank the selection committee of the NAA for awarding me the Ray Jewell silver medal which I shall always treasure.

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Mint and Die Parameters by Matching P.CREPVSI Coin Evidence and *MintSW* Meta-coinages

Pierluigi Debernardi¹

Abstract

*This work estimates the lifetime of the dies and the number of workstations as well as other parameters employed by the Roman Republican (RR) mint in its production of the denarii of Crepusius.² It does this by matching the coin evidence in terms of die links and symbol patterns with their corresponding software values. These estimates are not otherwise attainable with direct numismatic approaches.³ It demonstrates superior results to earlier investigations into this much-studied series. To reach this goal, a software model called *MintSW* has been developed that simulates mint operations which employed numbered dies with unfixed pairing. It was successfully tested on a small series of Fabius Pictor⁴ (RRC 268/1) and then applied to the Crepusius series.⁵*

Key words

[Roman Republican mint] [Roman Republican coins] [Die lifetime] [P. Crepusius]

Introduction

The fact that no consensus has been reached on Roman Republican (RR) mint operation and estimates of average die productivity for RR denarii shows the difficulty of this undertaking. Many scholars have addressed the problem, each arriving at different statements about die productivity,⁶ number of workstations and speed of striking. This is well illustrated by the extensively studied issue of Crepusius (RRC 361/1), dated by Crawford to 82 BC,⁷ which has often been used as a case study for die statistics and

1 I thank Richard Schaefer for his kind assistance in improving my English expression in the paper and for passing me his unpublished die catalogue reported in the Appendix. Without him, this work could not have been done. I also thank the Managing Editor for his substantial work editing the paper.

2 RRC 361/1, P.CREPVSI, is referred to as Crepusius in this paper; RRC stands for Crawford, 1974.

3 In a future paper, I will estimate die productivity by a standard numismatic method. If these two independent methods, standard and model *MintSW*, yield similar answers, the results can then be used with considerable assurance to calculate the output of the RR mint.

4 Crawford, 1965.

5 Both the model and the implementing code are named *MintSW*. The code was written from scratch on MatLab platform. The code is available on request from the author, noting that *can be applied only to series that features the same control mark scheme*.

6 For a thorough review of ancient die production see e.g. Callataÿ, 2011.

7 We prefer the earlier date of 83BC, discussed in depth in Debernardi et al., 2018 and Debernardi et al, 2020.

mint operation.⁸ Other two large RR series have also been die studied: Bursio,⁹ and Piso.¹⁰ However, Crepusius has benefitted from studies by four numismatists,¹¹ and because its unique system of control marks makes the number of obverse and reverse dies obvious. These two factors have led to its choice for modelling by a new software model developed by the author called MintSW.

A decade ago, Witschonke estimated mint parameters using exactly the same data presented by Buttrey 35 years earlier.¹² Both Buttrey and Witschonke aimed to determine quantitative information from a detailed investigation of Crepusius' coinage. It is worthwhile summarising their results. According to Buttrey, two workstations produced Crepusius and the three other issues of the triumvirate RRC 362-364. Since he estimated for that triumvirate about 2400 dies,¹³ and assumed 320 working days, he concluded that two die pairs per day were consumed at each workstation. Witschonke calculated this from a different perspective and with a different methodology. Assuming a production of 4000-5000 coins per day and a die productivity of about 20000 denarii, he arrived at a die lifetime of about four-five days.¹⁴ He then adopted the same 320 days assumed by Buttrey and concluded that a workstation consumed 65 to 80 dies/year. He did not take the next step, namely that the 1200 obv. dies of Buttrey would have required $1200/(80-65) = 15-18$ workstations. Therefore, starting from the same numismatic evidence, the number of workstations varies by a factor of 9: 2 for Buttrey; 18 for Witschonke. Buttrey assumed two workstations, because the dies divide themselves into two groups: fine style and gross style. Then he used this datum along with the total number of dies to estimate mint parameters. On the other hand, Witschonke assumed an average die lifetime and striking speed. Technically, this is called as an 'ill conditioned' problem; with the considered parameters there is no unique solution. Similarly, a system of two equations and three unknowns has an infinite number of solutions.

8 See all of Carter's articles on the topic; a selection is presented in the bibliography and in Esty, 2011.

9 DeRuyter, 1996.

10 Hersh, 1976.

11 Hersh, 1952, Buttrey, 1976, Witschonke, 2012 and still in progress with Richard Schaefer: http://numismatics.org/authority/schaefer_richard. Hersh's paper appeared in 1952, so in 2022 we celebrate its 70th anniversary. We owe to Charles Hersh an understanding of the sequence of the mint marks on Crepusius coinage. His work was taken over by Ted Buttrey, who continued adding pieces and discovered several new dies. Moreover, he also tried to infer some information about the mint operation, as the title of Buttrey, 1976 states.

12 Witschonke, 2012.

13 Including obverse and reverse dies. Actually, there were more. According to Schaefer's current die study of RRC 360,361,362 and the die study of Censor (n.5), the counted dies have reached about 2500.

14 This also agrees, as he notes, with Giles Carter's plot (Fig.1 in Carter 1983), yielding an average die usage of 50 hours. However, no detail is provided as to how that distribution is related time, as the statistical derivations and their connections to die studies are not a function of time.

To solve the problem of workstation number from genuine coin evidence, in recent years, a new investigation tool has been proposed and developed based on die charts.¹⁵ Starting from die studies, it provides an estimate of the number of workstations used in a coinage. It would be interesting if some of the experts with this technique would deal with Crepusius, using the full dataset provided in Appendix 4, Table A4.2, and see if my results are confirmed.

The previous discussion has shown the difficulty of extracting from die studies the quantitative parameters of mint operation. The common method to determine mint parameters is quantitative. It relies on the known quantity of coined silver and the known number of dies.¹⁶ From these data we infer the average die production. However, this simple and, in principle, solid approach is hard to apply due to the lack of precise data on coined silver in ancient sources. Even in the well-known case of the Amphictyonic League, there is uncertainty that leads, at best, to a range of plausible values. For Republican coinage, no quantity of silver is ever reported in ancient sources, so we have to rely on the rare occasions when they provide details about military forces in the field.¹⁷ I will discuss the available data in a forthcoming paper but can state now that there are difficulties in inferring the quantity of denarii. We have no clear statement in the sources about the cost of a legion, for which we can only make an educated guess. Also, we lack other important information, such as how often a soldier's salary was paid.

Therefore, in this paper I propose a completely different approach, based on a mintage model which I show can accurately replicate the numismatic evidence. At that point I determine from the huge sample of Crepusius coinage the related mint parameters, such as number of workstations and die lifetime in terms of workdays. In fact, the different die pairs are formed at the moment a die breaks or when starting a new work shift, details which are studied in detail by the *MintSW* model, because this is the only mechanism related to time that can be inferred from numismatic evidence.¹⁸

The new approach can be compared with Carter and Carter, 1982, but although starting from similar ideas, our aims and results differ. Carter and Carter focused more on die statistics than die pairs, die box refilling policy and corresponding die chains which all govern mint operation. Their work was based on a significantly smaller sample of Crepusius coins. With better graphical tools (the figures presented here), I provide detailed investigations of new parameters, such as the partial randomness of reverse die selection and the die box refilling policy.

15 Carroccio 2011, Bracey 2012 and Bracey 2017

16 See for example Molinari 2003 and Marchetti 1999.

17 *RRC* attempts this approach for the coinage of Annius, which is an example of its weakness. In fact, in this case, the period and the involved military force are not so well defined.

18 Esty, 1990 addressed the topic of die pairs, but under a different, more theoretical, perspective.

The Crepusius coinage has attracted much attention, as Carter's bibliography shows, because both its reverse and obverse dies are control marked. Thus, their pairing can be studied in all details. This essential information has not been considered so far but is the only way to determine the die lifetime in terms of mint days.

The one unknown still lacking to estimate die productivity is the quantity of silver minted per day. We can guess it with some accuracy by using the experimental archaeometallurgical work in Melle (10-12 coins per minute, or 600-720 coins per hour),¹⁹ complemented by Witschonke's model. It supposes a fixed quantity of coined silver per day in terms of pounds, which was arguably the approach required to guarantee safety and control of the precious metal in the mint. A pound of silver equates to 84 denarii in the late RR period, which allows approximately to 70 to 100 pounds per day as the possible range of production. In fact, the most reasonable range of 10 to 12 net work hours per day results in a minimum of 6000 and a maximum of 8640 denarii per day, equivalent to 70-100 pounds per day. This is more than the 50 pounds per day proposed by Witschonke but is in better agreement with recent extensive minting experiments.²⁰

It is difficult to find a good balance between technical *MintSW* details and the related numismatic information. *MintSW* is a software (SW) model developed by me and newly presented in this article to serve RR numismatics, by simulating mint operation. It was first applied to the coinage of N. Fabius Pictor (*RRC 268/1b*) in Appendix A3 because it is a small and well understood series perfect for testing the *MintSW* features. However, it cannot be used to determine quantitative information because its small number of dies makes it too prone to statistical variations.

19 Faucher, 2012, with 20 didrachms produced in 110 seconds, i.e. one each 5.5 seconds, or 11 per minute.

20 See also Sellwood 1962, Rottinghaus 2007, Faucher, 2009 and Faucher, 2016.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	
1 Nil	Nil					9																																	
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2 Fulmen	D	12				1																			3														4
2 Fulmen	E																																					13	
2 Fulmen	F																																					3	

Fig. 1: Upper corner of the pair matrix of *RRC* 361/1; the columns refer to the reverse dies, the rows to the obverse dies, the numbers at their crossings are the number of known specimens. This table updates Buttrey’s corpus with our latest data. Five of the denarii could not be verified by images. Currently, the table includes 3820 denarii comprised of 414 obverse and 453 reverse dies. Of these dies, 23 obverses and 51 reverses are known by only one example.

Crepusius: a data mine for mint operation parameters

I have applied MintSW to *Crepusius*, because it is such a large issue that statistical variations should average out. It is worthwhile starting with a brief summary of the features of the *Crepusius* coinage. Richard Schaefer kindly provided me with the data arranged in a spreadsheet, in the same format as that used for *Fabius Pictor* (Appendix 3).²¹ The table is, however, with about 500 rows and columns, and too big to show in full here. Just the left top corner of *Crepusius*’ table is shown in Fig. 1 by way of example. In Appendix 4, I present the die catalogue derived from the table.²² The benefit of *Crepusius*’ control mark system is that it orders the dies, which, at least in part,

21 This avoids the necessity of providing two files, as in the case of Buttrey, where he sorted the obverses in one and the reverses in the other, indicating all the couplings. Buttrey’s archive, including both catalogues and images, is in the Fitzwilliam Museum.

22 The results of this paper are based on a 2020 matrix update. We take the opportunity to present in Appendix 4 a catalogue updated on 15 Jan 2022, where another singleton disappeared and another 46 new specimens are added, reaching 3866. This does not modify the results presented here with a former dataset.

correspond to the order in which the dies were used.²³ Therefore, even the missing dies can be counted.²⁴ The reverse dies have a running numeral, while each obverse die has one of 25 symbols plus one of the 21 letters of the Latin alphabet. Since the last symbol (Barley) is known only with letter A, the programmed number of obverses is 505 (24 x 21 + 1). The number of reverse dies is given by the highest observed numeral, 519. Thus, the number of obverse and reverse dies are balanced. Of course, not all the dies have reached us. While the missing reverse numerals can be attributed, quite securely, to early die breaks, for obverses the situation is different. In fact, there are two symbols, Thyrsus and Altar, which seems to purposely lack most of the letters. Thyrsus (Buttrey's symbol no. 9) has only A-F and Altar (no. 20) has only A-G. This might be explained as a way to synchronise obverses and reverses, with the aim of reaching all the 25 planned symbols.²⁵ As support for this, I note that the two short-lived symbols fall at regular positions— 9 and 20. Therefore, the engraved dies²⁶ are 505-29=476 obverses and 519 reverses. These are the values entered into MintSW (see Table 1).

In Fig.2 the original Hersh chart is compared to a modified one, which shows a small square at any x-y position where a pair is found in the updated catalogue. As can be noticed, progress has been made in 60 years, but the general appearance does not change much. Similar plots are shown later, and they will be termed a 'Hersh chart'.

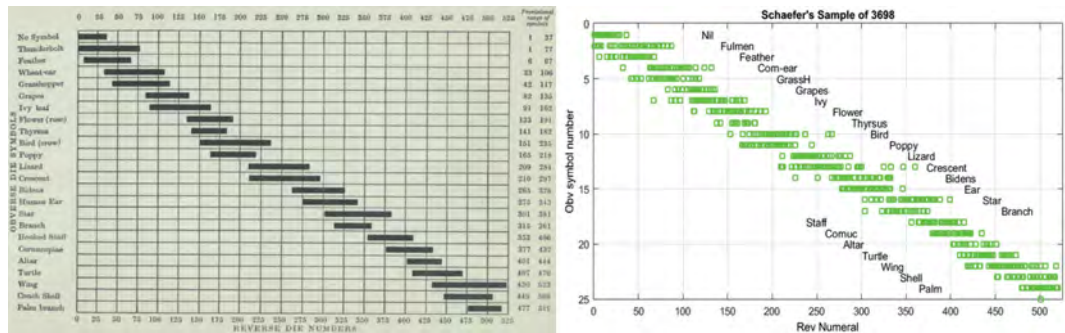


Fig. 2. Left: Hersh chart (Hersh, 1952), a simplified matrix representation of the Crepusius coinage. Right: similar but with all the actual pairs indicated.

Before starting to apply MintSW to Crepusius, it is useful to list all its parameters in Table 1, their best fit values, and provide short descriptions. Parameters no. 1 and 2

- 23 This does not occur in series like L.PAPI, where the control-marks are strict pairs of objects. Since pairs are unique and no variation is known, all the relevant information that can be gained from series like Crepusius is missing.
- 24 See Esty, 2011 for his model applied to Crepusius reverse. With the updated Schaefer's data, one gets 511 reverses (vs. 519 of the highest numeral) and 440 obverses. The latter compares worse with the theoretical number of 513 (symbols x letters), even if one does not include the 15 missing dies that form a gap in the observed dies.
- 25 A strange explanation is proposed in Müller, 2006
- 26 There are a handful of dies that duplicate letters and numerals. But these are clearly engraver mistakes and are so few that they need not to be taken into account.

are the engraved dies for the series. These numbers are the only ones to come from the Crepusius data. Nos. 3 to 8 are determined as the best fit to the Crepusius data. For nos. 4 and 5, refer to the Appendix; in brief, they determine the decay of the die lifetime which is assumed to obey an exponential law. This is a common and reasonable assumption, also used in today's industrial models for reliability. No. 6 makes this model more realistic, describing a maximum die use to guarantee a good quality of coinage. Nos. 7 and 8 govern the die box managing, which is of great importance to achieve a good matching of the die pair patterns (see Appendix). FAdd is the number of added dies to the die box per workstation W ; the total number of added dies to the die box is therefore: W (no. 3) \times FAdd (no. 7). FRand describes the randomness governing the die pairs from one day to the other. Nos. 9 and 10 are inner model parameters. No. 9 allows computing of an average among SW coinages and also, and very importantly, to compute standard deviations of the various quantities. No. 10 determines the size of the SW coinage. In order to limit computation time, this must be larger than the numismatic sample, but at the same time much smaller than the original production.

No.	Parameter	SW name	Unit	Value
1	Produced Obverse Dies	D_O	-	478
2	Produced Reverse Dies	D_R	-	519
3	Workstations	W	-	5
4	Die Lifetime Obverse	DLT_O	Days	1.50
5	Die Lifetime Reverse	DLT_R	Days	1.35^{27}
6	Maximum lifetime of the die	DL_{max}	Days	3
7	Die Addition Factor to Die-Box	FAdd	-	5
8	Randomness Factor	FRand	%	20%
Subsidiary variables				
9	Number of SW mintages	NMintages	-	20 to 100
10	Number of SW strikes per shift	NSWS_1	-	20

Table 1. Table of *MintSW* parameters of Crepusius simulation. DLT_O and DLT_R are related by R/O ratio (1.1).

To illustrate how the random features of the mint process are investigated, I present in Fig. 3 some relevant *MintSW* results such as the computed obverse and reverse die numbers, number of pairs and corresponding singletons²⁸. To show the much higher complexity of a coinage as large as Crepusius, in comparison to Fabius Pictor, examples of die reservoirs are provided at the top of Fig. 3. They comprise more than 12,000 SW strikes, the result of summing the varying strikes associated with each die. Below of the

²⁷ The obverse to reverse lifetime ratio is directly input from the ratio of counted dies, nearly equal to 1.1. See A1.3.

²⁸ A die singleton is a die that is known in only one specimen. Similarly, a die pair singleton is an obverse/reverse combination known in only one specimen.

reservoirs, the results of 20 SW mintages are shown, together with their average values, indicated by a line.

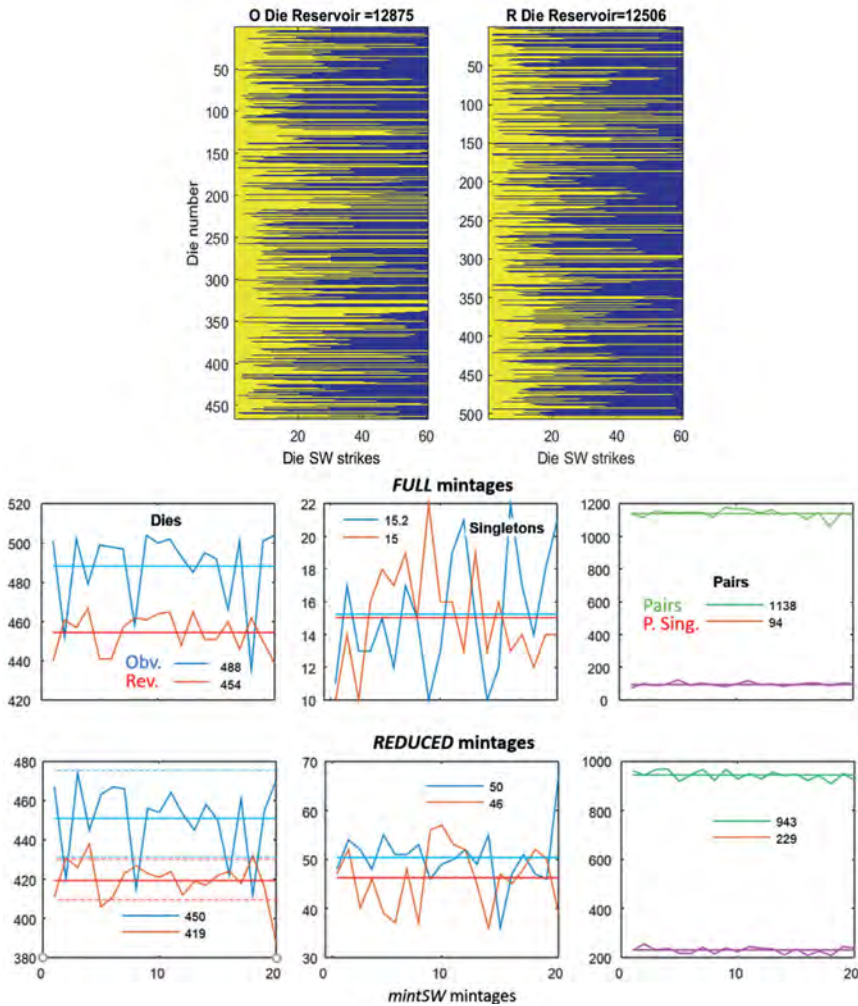


Fig. 3. Top: examples of die reservoirs for Crepusius ($DLT_0 = 1.5$, $DL_{max} = 3$, $NSWS_1 = 20$). Only the dies with at least 1 SWS are included in the plot. **Below:** Statistical analysis of the number of dies, die pairs and singletons of the same 20 *MintSW* mintages of Fig.5; the results of each SW mintage are reported together with their averages (straight lines), specified in the plot legends. The first row refers to the full SW mintages (all featuring more than 10000 coins, see titles of the die reservoir plots), the second row to samples reduced to 3700 coins, the size of the actual numismatic sample. Here and throughout this paper, O or Obv = Obverse, R or Rev. = Reverse, Sing. = Singleton and P or Pairs = Die Pairs.

To avoid overcrowded plots, in one case (the lower left plot) the corresponding standard deviation confidence intervals are reported by two dashed and thinner lines. 85% of the samples lie within these lines. Fig.3 also illustrates the concept of sample reduction.²⁹ By comparing the two sets of results, full and reduced (top and bottom plots), different

²⁹ See Appendix 2.4.

values are observed. All the quantities of the reduced sample are lower, except the singletons, which, as expected, are more numerous. In the same way, we could define reduced samples of different sizes, apply the Esty formulas and test their validity.³⁰

Number of workstations for Crepusius

In this section I discuss the most probable number of workstations used to strike the coinage of Crepusius. Buttrey thought that two workstations were used. To verify that, we must quantify the overlapping of the obverse symbols and reverse numerals. For each obverse die symbol (numbered from 1 to 25), Fig. 4 (left) presents the range in reverse numerals (NumDiff), their mean value and their standard deviation. Similarly, for each reverse die number, Fig. 4 (right) presents the range of obverse symbols (SymDiff), their mean and their standard deviation.

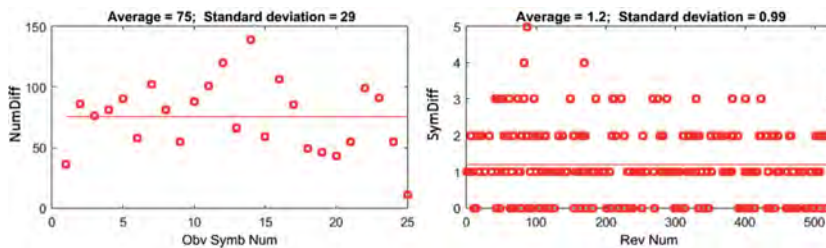


Fig. 4. Left: For every symbol number, its rev. numeral range is reported. **Right:** Similarly, for every reverse numeral, its obverse symbol number range is provided. The averages and the standard deviations are given in the plot titles.

The same quantities are then computed for the *MintSW* coinages and compared to the actual numismatic values, as shown in Fig. 5 where *NMintage* is set to 20. This subsidiary parameter allows computing averages and standard deviations of the investigated parameters. Every mintage results in an average and a standard deviation of *NumDiff* and *SymDiff*, which are compared to actual data. The values in the plot titles of Fig.4 fit well the corresponding values in Fig. 5. This good fit of the actual and SW sample supports the adopted set of model parameters. The number of workstations is not trivially related to the Hersh charts. In fact, the superposition of different numeral ranges depends also on the number of dies in the die boxes. While in *RRC 268* only one style occurs, in *Crepusius* there are clearly two styles, corresponding to engravers whom Buttrey names F (Fine style) and G (Gross style).³¹ In this large coinage, the computed engraving rate also has to be monitored and, therefore, is reported in the plot titles (*Engraver Dies/Day*). It has to be compatible with the engraving rate of 2-3 dies per day obtained by modern experiments on ancient coinages. From Fig. 6 we can conclude

³⁰ This is out of the scope of this paper and might be treated in a future work. Nonetheless, it is useful to point out that *MintSW* might be used to go beyond Esty simple model, allowing to determine not only the engraved dies, but also other mint parameters.

³¹ This number most probably must be doubled to account for the engraving of the reverses.

that the number of workstations cannot be derived with certainty, but a range can be inferred.³² Fig. 6 shows the results for two, five and eight workstations. Two workstations do not match the symbol and numeral overlaps of the Crepusius data, while eight workstations yield overlaps that are too large. Moreover, by looking at the number of needed dies per day, two workstations (Anvil in plot titles means workstation) could be fed by just one engraver (around 2 obverse dies per day), while 8 workstations would need more than two engravers, requiring an engraving speed of nearly seven dies/day.

In Fig. 7 the most probable number of workstations (4-6) is used to produce corresponding Hersh charts. Five workstations best match the Crepusius data,³³ but four and six cannot be excluded. Other parameters, not considered here, could refine this parameter. For example, if the target production for the year were known, more could be done and/or a crosscheck could be made.

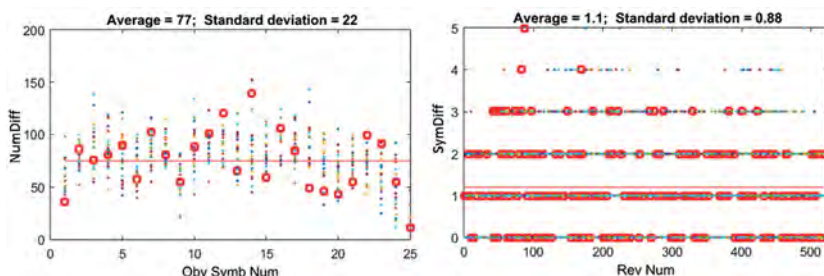


Fig. 5. As in Fig.4, but the actual data (in red) are superimposed with 20 *MintSW* mintages (one for each coloured dot); thin red lines correspond to Crepusius averages. In the plot titles, the average over mintages of averages and standard deviations of the two investigated quantities is reported. From the above plots it results that one cannot focus on the details of each pattern (fully governed by the random features of the die breaks and, to a lesser extent, die exchanges at the next shift), but just on their major statistical features— averages and standard deviations.

³² The three blue plots in Fig. 6 are just one of an infinite number of SW mintages, as discussed so far, and were not selected to achieve a better or worse result. Any time this same simulation is run, a different result occurs.

³³ It is interesting to point out that most probably 5 workstations produced the main body of L.CENSOR (*RRC* 363/1), as recently discovered by its die study, Debernardi-Campana-Lippi, 2020.

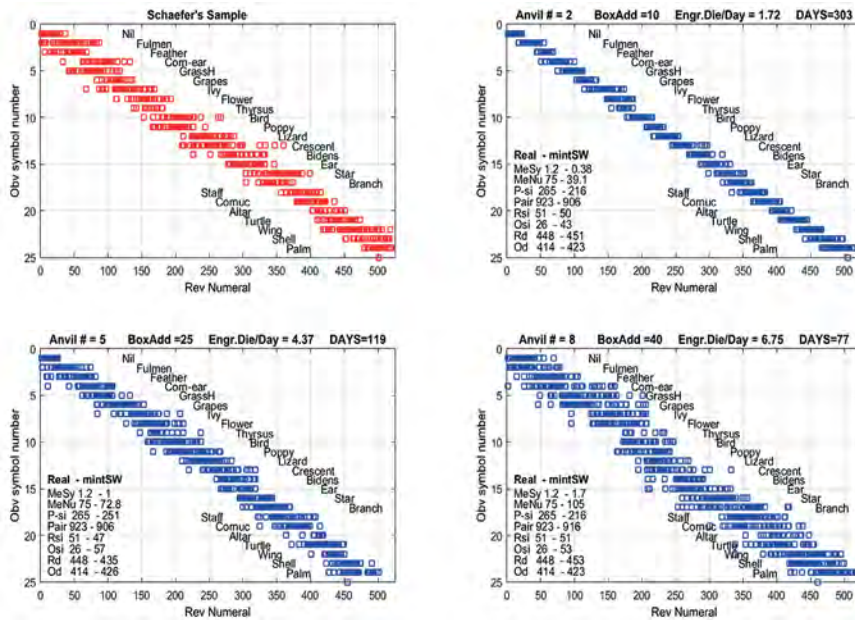


Fig. 6. Comparison of the modified Hersh charts for actual and *MintSW* samples obtained for different workstation numbers: 2, 5 and 8. A variation of W implies a corresponding increase of the dies added to the die box ($BoxAdd$) when getting empty, as indicated in the titles. In the left-lower corner of each plot some of the relevant mintage parameters are compared to those of the numismatic sample. The first two data refer to the quantities in Fig. 4, followed by $P-si$ = die pair singletons; $Pair$ = die pairs; Rsi = reverse die singletons; Rd = reverse dies; Od = obverse dies.

The full parametric deployment of *MintSW* is presented in Fig. 8, where I investigate the effects of W and $FAdd$ (workstations and die box refill factors) on the two parameters ($NumDiff$ and $SymDiff$) that quantify the spread in the Hersh charts (Fig. 6 and 7). One hundred mintages have been run, in order to better determine the averages and standard deviations. $FAdd$ is varied on the horizontal axis, and results are shown for three values of W . To give an idea of the simulation time, a mintage of 10-12000 SW coins takes about four seconds on a standard PC. Therefore the simulations in Fig. 8, comprising three values of W and 7 of $FAdd$ (i.e., 21 combinations), repeated 100 times, requires about two hours.

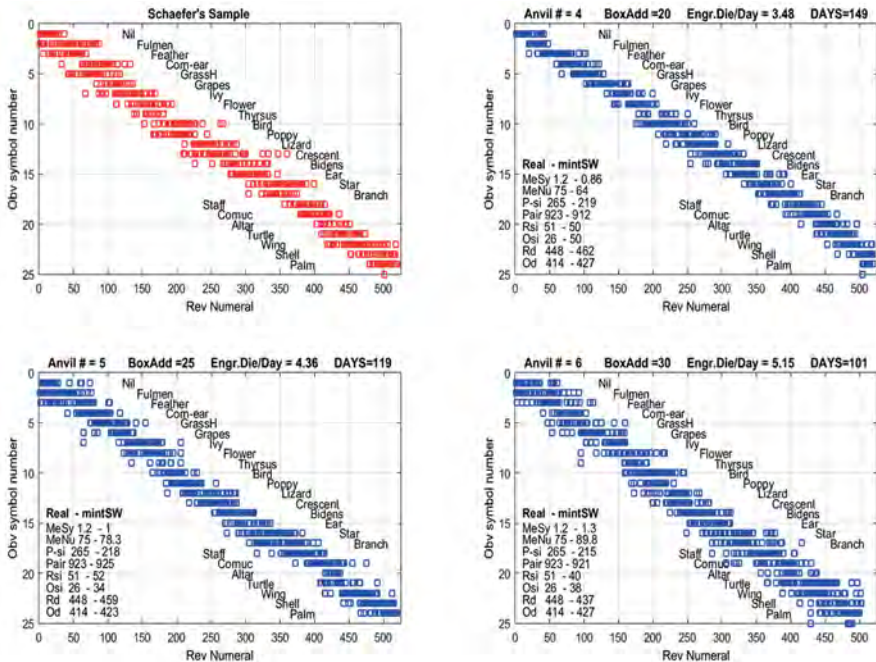


Fig. 7. As in Fig. 6, but for W=4, 5 and 6. However, the W=5 plot is for a different SW coinage than in Fig. 6; compare the MintSW data of the two mintages.

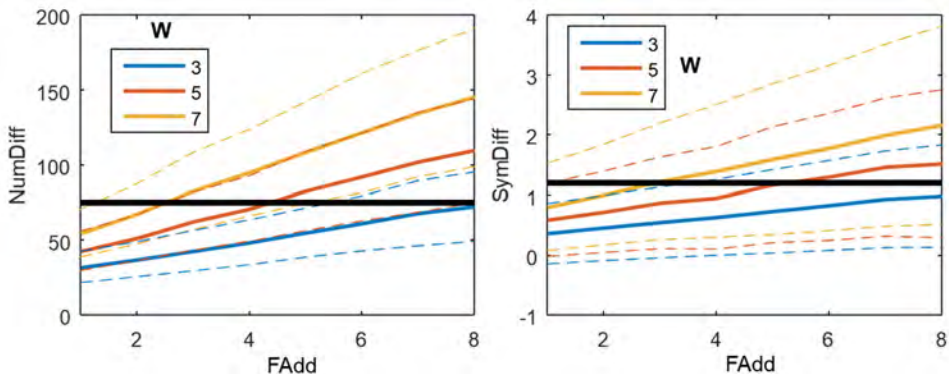


Fig. 8. Parametric variation of FAdd for 3 values of W (workstations, see legend), and their effects on NumDiff and SymDiff (see Figs. 4-5). Thick black lines show the Crepusius data. $DLT_0 = 2$ and $FRand = 20\%$.

The continuous coloured lines report the averages over 100 SW mintages, while the thin dashed lines correspond to the average \pm the standard deviation. They define regions comprising 85% of the SW mintages (see also Fig.3 and related comments), which is very important information considering the randomness of the problem. The parameter values that make the numismatic and simulation results coincide are similar for both NumDiff and SymDiff, indicating MintSW is consistent. FAdd of 8.5, 4.3 and 2.5 for three, five and seven workstations (respectively) confirm that five workstations provide the best compromise. In fact, it would be strange to require 8.5 dies per workstation

with just two workstations involved when the Fabius Pictor simulations suggest a value of four (see Appendix). For opposite reasons, 2.5 seems too low. Interestingly, $\text{BoxAdd} = 21 = 5(W) \times 4.3$ matches the size of a symbol die set of 21 letters.

Five workstations is the first relevant *MintSW* achievement. This number substantially differs from Buttrey's guess of two, which forced him to require the consumption of two pairs of dies per workstation per day, reducing the die productivity to around 4000 coins/die.³⁴

Die pair randomness (FRand) and pair numbers

FRand has not been discussed in any detail so far. It was added in the model at a second stage, as a feature needed to improve the unsatisfactory results on the die pair number statistics after preliminary tests. In fact, in *MintSW* first version, at the end of every shift the dies were assumed to go back into their die boxes and, at the next shift, chosen fully at random from the die box.³⁵ The result of this assumption is shown by the blue curves in Fig. 9, where the DLT_O (die lifetime) is on the horizontal axis and FRand is varied parametrically. A match of the numerical results with the Crepusius data occurs only for a too short DLT_O of about 0.5.³⁶ None of the other *MintSW* parameters (see Table 2) can bring the number of pairs in line with the Crepusius figures. The only solution is reducing FRand to 100%, whose foundation is described in Appendix A1.5. FRand=100% means that all the dies are put back into the die boxes at the end of the work shift, and the dies for the next shift are randomly selected from their boxes. At the other extreme, FRand=0% means that a die, when selected, is used until the end of its life. The results of Fig. 9 condemn both these two extreme conditions. Denying any randomness (FRand=0%) yields too few die pairs. Even FRand = 5% never provides the 923 actual Crepusius pairs. On the other hand, FRand = 100% would imply a DLT_O as low as 0.5, which in turn yields a die productivity of around 4,000 coins per die, which seems too low.³⁷ Instead, values of FRand in the range 10-30% results in a good match of the computed pairs to the Crepusius data, corresponding to a **1-2 DLT_O range**.

FRand of 10-30% works well for the following reason. The obverse dies were probably blocked into their anvils, and the reverse dies into their hammers, then left there from one shift to the next. Most probably therefore, once selected at random from their boxes,

34 Assuming 12-hour shifts and a maximum minting speed of 720 pieces/hour. This maximum is suggested by Thomas Faucher (private communication, Oct. 2018), compared to the rate of 650/h achieved by his modern minting team of two people (Faucher, 2009, p.65).

35 This corresponds to force FRand to 100%.

36 Which would result in the value postulated by Buttrey, resulting then in the problems just described.

37 Not only all the estimates provided so far, by different approaches, point to values ranging from 15000 to 30000, both in Greek (see Marchetti, 1999, Callataj, 2011) and Roman RR coinage (RRC p. 694) but also the results of modern minting experiments (see Sellwood, 1963 and Faucher, 2012) point to values higher than 10,000.

the dies were used in their holders until they broke.³⁸ The residual 10-30% randomness is to be ascribed to the possibility that the die holders could be exchanged among the different workstations on the next shift.

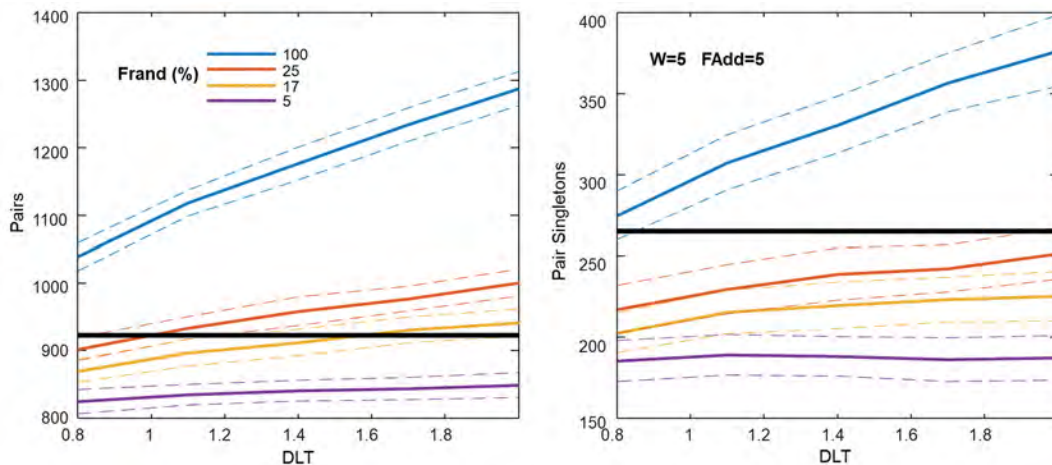


Fig. 9. Parametric investigation by varying DLT_O in a reasonable range for four values of $FRand$, as shown in the legend, and their effects on the number of pairs and their singletons. Black thick lines report the numismatic values. The thin dashed lines show the confidence intervals.

Dies and Pairs Frequency Distributions vs. Lifetime of the Dies

This section explores another die study parameter— Die Frequency Distribution (DFD). DFD sorts the dies according to their multiple occurrences, also referred as frequencies.³⁹ While DFD is a well-known analytical tool, to date no attention has been paid to die Pairs Frequency Distribution (PFD), disregarding the fact that coins represent die pairs.⁴⁰ Indeed, in this paper DFD and PFD prove their fundamental impact on my analyses. As a first example, in Fig. 10 the actual DFD (at left) is compared to those of 50 *MintSW* coinages. Note that the plots use a logarithmic scale to encompass both large and small numbers. The actual numbers can be obtained by using the graduated hashmarks on the vertical axis. The SW data appear as ‘dot clouds’, which should agree with the actual data when the adopted model parameters are well chosen.

38 Vermeule, 1954 and Malkmus, 2008.

39 Albaredo et al., 2021.

40 Die pairs should be an essential part of any die study and die chart investigation (Carroccio, 2011 and Bracey, 2012, 2017). They have never been treated by statistical analysis as have the dies themselves (Esty 2011). In issues with fixed die pairs, like L. Papius (RRC384) or L. Roscius (RRC412), the statistics coincide. In *Crepusius*, the formation of pairs is a two-dimensional random process, governed both by die breakage and random selection of the dies from their boxes.

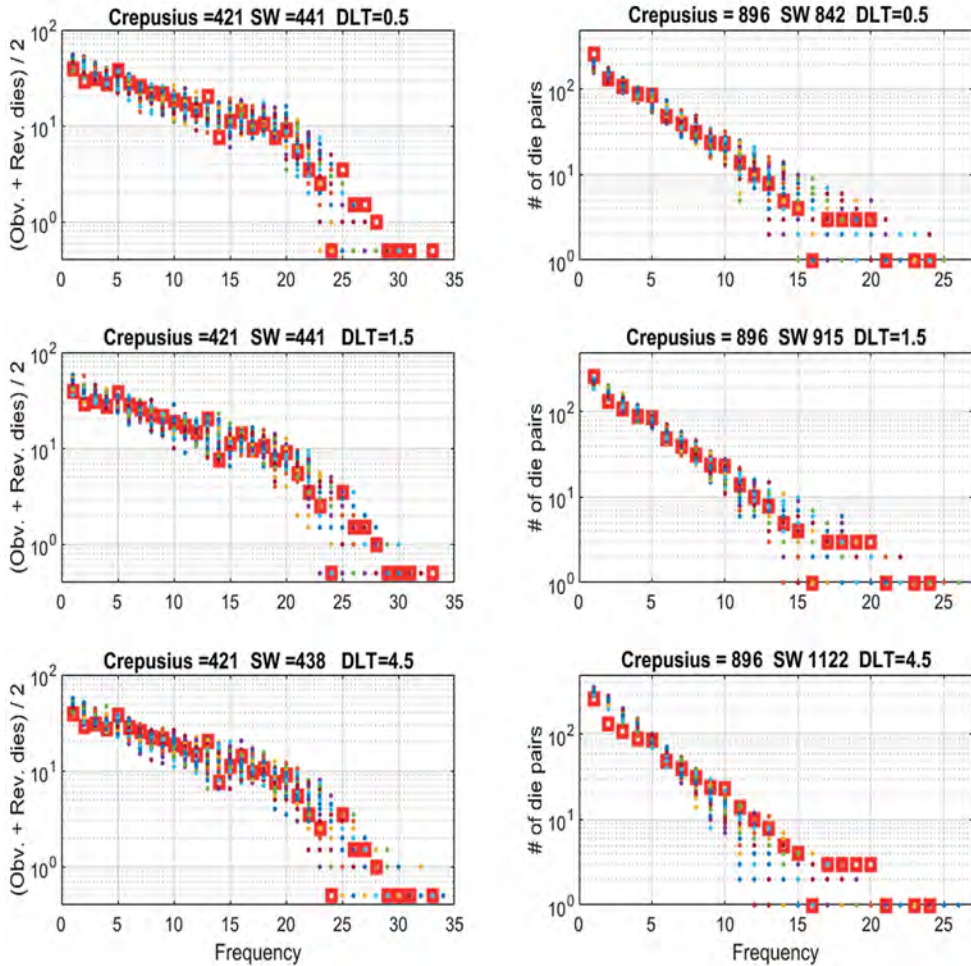


Fig.10. On the left, 3 frequency plots of the average number of obverse and reverse dies. On the right are similar graphs for the number of die pairs. $DLT_o=0.5$ on the top row, 1.5 in the middle, and 4.5 at the bottom. DL_{max} is set to $2 \times DLT_o$. The Crepusius values are shown in all 6 plots by red circles. The plot titles give the Crepusius and MintSW values. In Fig. 12, bottom plots, obverses and reverses are reported separately.

DFD is independent of die lifetime (DLT_o), but PFD is clearly dependent on it (see Fig.9 and Fig.10). The difference in dependency is due to the following facts. Changing DLT_o does not affect DFD, because everything scales accordingly. DLT_o affects the die's coin production, but not its frequency statistics. On the other hand, as we learnt above, the die pairs depend on DLT_o . A shorter DLT_o reduces the number of die pairs, because they have fewer chances to form. Conversely, a longer DLT_o allows more opportunity for die pairs to occur. And as the number of die pairs changes, so does their distribution (PFD). Thus, a too low DLT_o generates too few singletons and too many die pairs at higher frequencies. One could say the 'dot cloud' is tilted counterclockwise. The opposite occurs at too high DLT_o (higher values at low frequencies and lower values at high ones), corresponding to a clockwise tilt. A satisfactory alignment is achieved

for intermediate values. Here $DLT_0=1.5$ is adopted, but values in the range 1.3-1.6 would also work well. A DLT_0 of 0.5 or 4.5 corresponds to Buttrey's or Witschonke's proposals, respectively. In Fig. 11 the variation of DL_{max} , the maximum lifetime of the die, is shown to produce the strongest effect on DFD. Very high values ($DL_{max}=30$ in the lower plots of Fig.11) approach the ideal case of a pure exponential die lifetime model, corresponding to an exponential DFD, i.e. a linear behaviour in logarithmic plots. Very low values ($DL_{max}=0.5$ in the upper plots of Fig.11) cause a strong DFD deformation, especially when DL_{max} is made coincident with DLT_0 . By truncating the die break probability distribution too early, the dies have no chance to populate the higher frequencies and certain intermediate frequencies experience a rise.

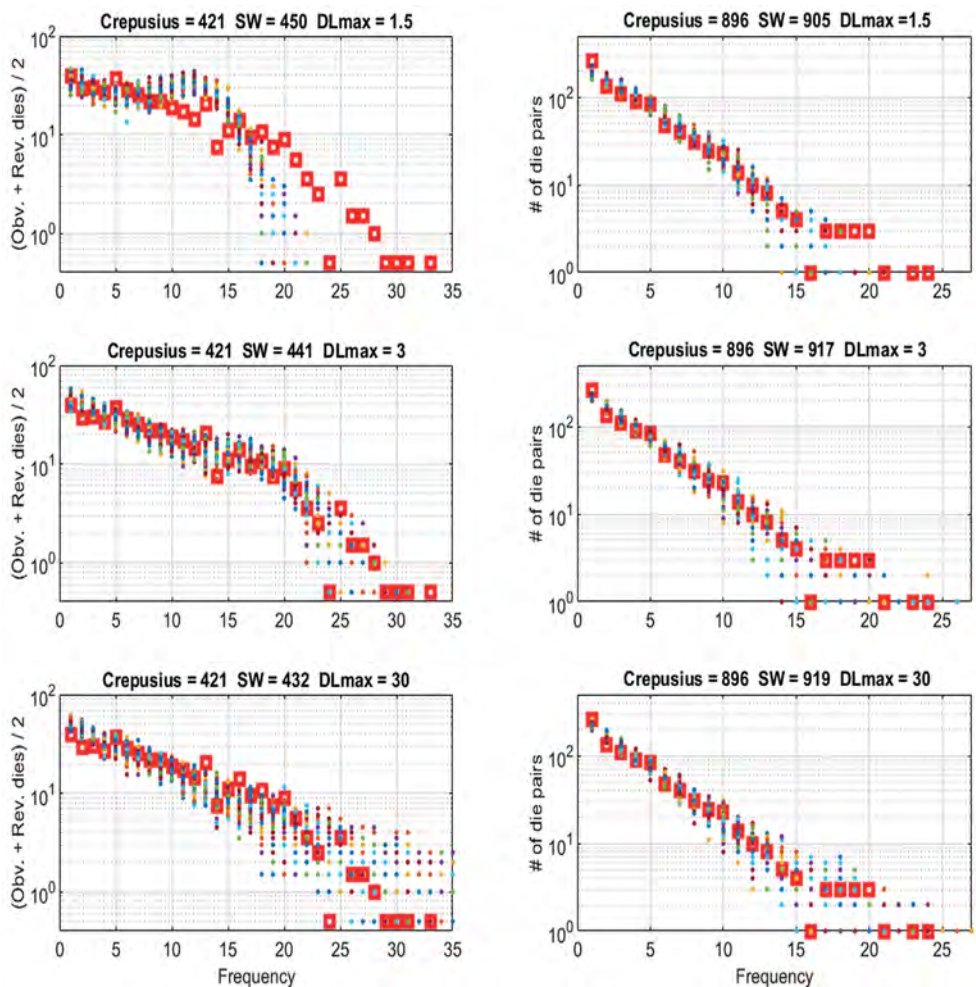


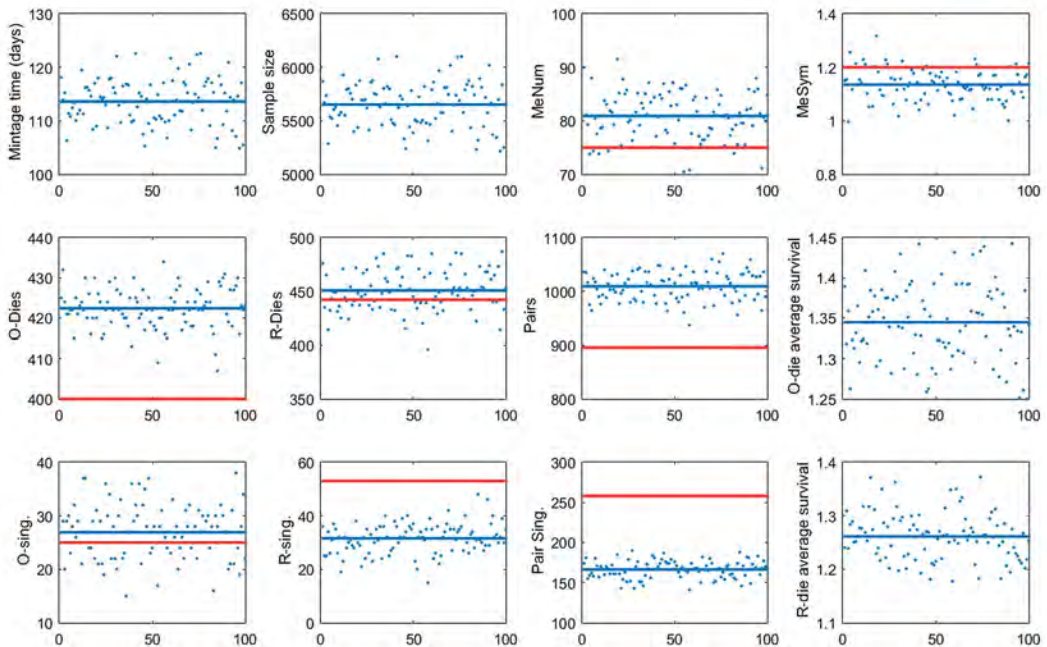
Fig. 11. As in Fig. 10, but here DLT_0 is set to 1.5, while DL_{max} is progressively increased from top to bottom.

This results in a sort of bump in the DFD plot, which in this case has its maximum at around 12. After this frequency, the decay is faster, causing a much earlier end of DFD (at

around 20 instead of 30). $DL_{max}=3$ (middle plots) results in a satisfactory comparison with the Crepusius data. They feature a steady decay in the first frequency range, which becomes steeper at around 20. The MintSW data well imitates this behaviour, which is caused by DL_{max} . Compared to the top plot, $DL_{max}=3$ (instead of 1.5) still causes a change of slope in DFD, but at higher frequencies and with a fainter bump at around 20.

In summary, I have discovered almost decoupled effects of DLT_o and DL_{max} . These two parameters govern die usage in frequency distribution; namely, DLT_o acts on Pair Frequency Distribution (Fig.10), and DL_{max} acts on Die Frequency Distribution (Fig.11). I used this property to determine their values by comparison to the Crepusius data and found that $DLT_o=1.5$ days and $DL_{max}=3$ days are good fits.

Plots of SW mintage results for *MintSW* full samples



(See Fig 12 reference on Page 43)

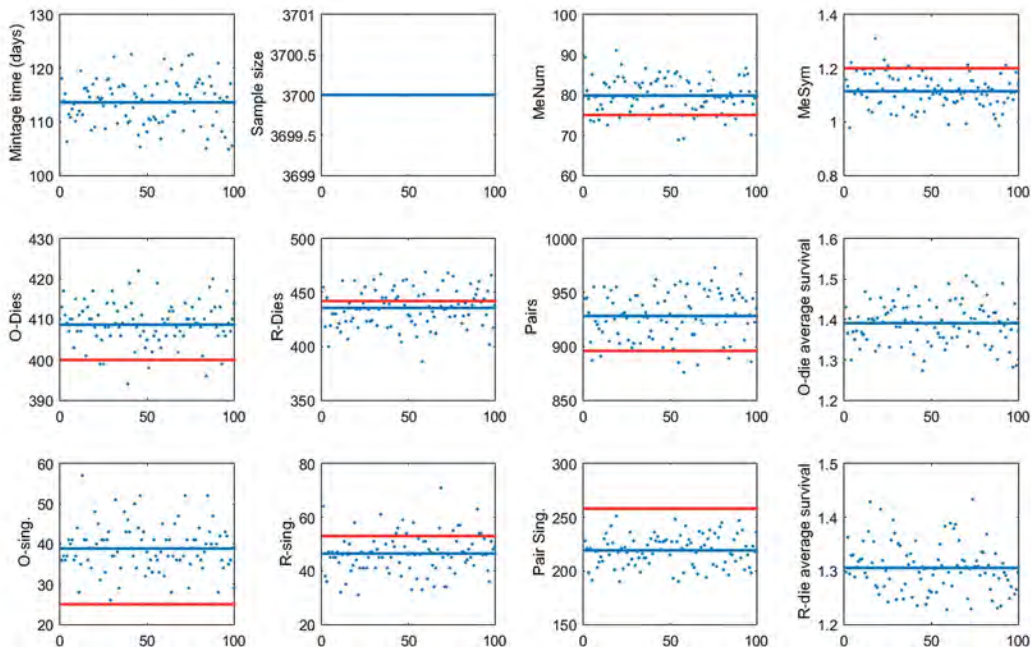
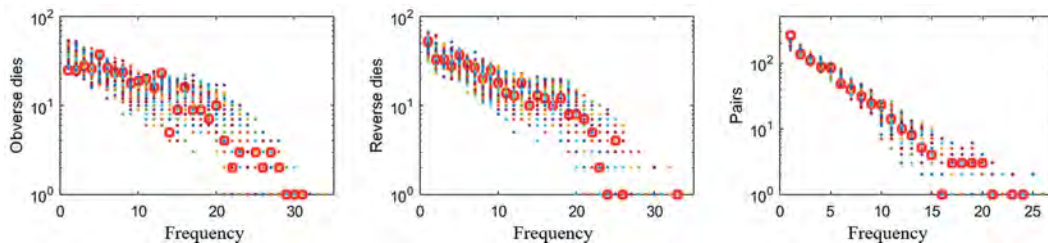
Plots of mintage results for *MintSW* reduced samples*MintSW* frequency plots of Dies and Die Pairs

Fig. 12 Various parameters, specified by the y-axis label, for 100 *MintSW* mintages (dots) which are plotted on the x-axis, using the parameter set in Table 1. MeSym = Mean obverse symbol number, and MeNum = Mean reverse numeral. The results for full samples are shown at the top, and for reduced samples in the middle. At the bottom, the corresponding frequency plots are shown. The red lines and circles represent the Crepusius data, and the blue horizontal lines the average over the 100 mintages. Mintage time and die survival are not known for Crepusius.

Overall comparison of *MintSW* and actual Crepusius coinages

In this section I provide some additional details and give a thorough overview of the mintage features with the adopted model parameters. Fig. 12 presents all the SW results corresponding to Table 1. Two other results, Mintage time and Die survival, unknown from numismatic evidence, are also shown. Fig. 12 is split into three sections, as the titles make clear. The first two sections refer to full and reduced SW samples, and the last section to the corresponding Frequency Distribution of dies and die pairs. In the first two sections, all the quantities are reported for every mintage and the corresponding averages are represented by horizontal lines. Once again, I stress that the Crepusius data

represent just one of the nearly infinitely different mintages that could have occurred in the production of that series. The Crepusius data, our only reference, is shown by red horizontal lines. The simulated results are reasonable if the red lines fall within the dot clouds. However, this is required just for the reduced sample constrained to the actual Crepusius data. I observe that the Crepusius data fall, with two exceptions, within the dot clouds. First, the number of Obverse Singletons is considerably greater in the simulations. Second, the number of Die Pair Singletons is higher in the Crepusius data. The obverse and reverse frequency distribution plots are presented at the bottom of Fig. 12, and the obverses show a different behavior than the reverses. The distribution is roughly flat for the first frequencies and shows a peak at frequency 5 but, overall, falls within the simulation dot cloud. The obverse die anomaly is also manifested by the high imbalance between obverse and reverse singletons, which in such a large sample with so many dies usually should not occur. For the rest, all is fairly consistent:

- The Hersh charts' average parameters fit satisfactorily, especially MeSym and MeNum
- The totals for the reverse dies (Fig.12 R-Dies) and reverse die singletons (Fig.12 R-Sing.) are satisfactorily comparable
- The number of die pairs is well reproduced.

Supported by the satisfactory agreement with all the Crepusius data, including dies and die pairs frequency distributions (the three bottom plots in Fig.12), the other MintSW results look trustworthy. One revealing and important parameter is the mintage time, which ranges between 110 and 120 days, nearly the same proposed by Buttrey, but with very different coinage outputs. In Fig.12 I observe average die lifetime values of around 1.3 days, resulting from $DLT_o=1.5$ days and $DL_{max}=3$ days. The average die lifetime corresponds to DLT_o only for infinite DL_{max} , and it is shorter for finite DL_{max} . Discarding the worn dies before they break shortens the average die lifetime by about 15%, which is one of MintSW's new results. Long lasting dies were discarded after three days, or ~20000 coins. These data are compatible with Faucher, considering the heavier weight of the tetradrachms, which require stronger strikes and correspondingly shorter die lifetimes. However, a very minor portion of coins were struck with old dies, and the average of 1.3 days, paired with 70-100 minted pounds per day, yields the die productivity sought: 7000-12000 coins/die.

Conclusions

In this paper a new computer model, MintSW, has been presented and demonstrated capable of reproducing the features of the most deeply investigated and statistically known Roman Republican issue - the series of Crepusius. All the information contained in its statistics has been exploited. In particular, a parameter never previously taken into account has been considered: die pairs. Besides governing the Hersh chart, when placed in a matrix arrangement die pairs are found to be of fundamental importance when

their computed frequency distribution is compared to their actual distribution. By varying the parameters that govern the mintage over reasonable ranges, and producing for each set 100 random SW coinages, the most likely values of these parameters have been identified as best fitting the numismatic evidence. In this way, the most probable values for the number of workstations, die lifetime and die box maintenance have been derived. Moreover, I determined their impact on parameters critical to production - the average die lifetime of 1.3 days and the corresponding average mintage of 115 days.

The use of all MintSW details governing the mint operation and the identification of the best fits to the Crepusius data allow for the first time the calculation of a self-consistent mint parameter set. Combined with the hammering speed suggested by modern minting experiments, this results in more precise quantification.⁴¹

About the author

Pierluigi Debernardi is an electronic engineer who graduated from the Politecnico di Torino in 1987. He is Senior Researcher at CNR, the Italian National Research Institute that conducts public research together with the university. His research in Vertical Cavity Surface Emitting Lasers, micrometer size semiconductor devices that populate modern cars and smart-phones, is renowned worldwide. He has published about 200 papers in this field. His 20-years interest in Roman Republican coinage has its main focus on the Second Punic War productions, with some exceptions, like the series produced in the middle of the II Cent. BC with XVI mark of value, and the Crepusius triumvirate productions. He has authored or co-authored about 40 papers in reputed numismatic journals, putting his technical skills at the service of numismatic research.

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41 In a paper in preparation, I will compare the achieved results to those that we can gain by studying the whole triumvirate in a more traditional way, deploying the die estimates based on die counts and a probable quantity of coinage for that year.

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Appendices

In these Appendices more technical details of MintSW are provided for the interested reader. No unrelated numismatic content is given, only details relating MintSW's use of Crepusius and Fabius Pictor statistics.

Appendix 1. *MintSW* inputs

MintSW's purpose is to model the mint operation and thereby determine the most probable values of its parameters and their confidence intervals. The first step is to define its input parameters and test them to produce results compatible with the numismatic evidence. The following set of parameters is considered:

1. Obverse and reverse dies totals, D_O and D_R ,⁴² are the number of engraved dies, not to be confused with the number known from surviving coins. D_O and D_R depended on the volume of silver to be minted and its estimate was surely among the skills of the mint-masters. D_O and D_R are generally unknown to us today but can be determined for those issues where the dies are uniquely numbered.
2. Number of workstations (W) mainly depends on the overall quantity of coinage to be produced.
3. Two Inputs for die survival and usage:
 - (a) Die lifetime (DLT_O)
 - (b) Maximum die lifetime ($DLmax$), for Obv. and Rev.

I assume that DLT follows an exponential failure law. This is only ideal, because it would allow an infinite life, even if with little probability. $DLmax$ makes the die lifetimes finite, accounting not only for breakage, but also the discarding of dies due to wear and damage.

42 In this paper shortcuts for the name of the variables of *mintSW* will be extensively used, as commonly is done in the scientific framework to speed up the narrative.

DLT_O and DLT_R stand for obverse and reverse die lifetime. Their ratio is a fixed input, computed from the numismatic evidence of the counted dies. In the case of Crepusius, 414 obverses and 453 reverses result in a 1.1 ratio, i.e., $DLT_R = 1.1 \times DLT_O$.

4. Diebox refill scheme: This is relevant, especially since the mint normally used multiple workstations. In numismatics a diebox conceptually is the pool of available reverse dies which are selected (usually assumed randomly) at the beginning of the workday to be paired with obverse dies, conceptually seen as fixed to their workstations.⁴³ The diebox is a concept, not a real physical box, frequently used in ancient numismatics. Michael Crawford used it to explain the pattern of letter pairs in the coinage of Fabius Pictor.⁴⁴ From modern experiments which engraved dies similar to ancient coinages,⁴⁵ it seems that an experienced engraver could produce up to three dies per day.⁴⁶ This refill rate needs to be taken into account. Since the die engraving proceeded in parallel with the minting, the refill rate must be compatible with the engraving speed. In *mintSW*, the refill rate is assumed constant and proportional to the number of workstations. The number of dies added to the die-box, the Addition Factor (FAdd), is to be determined by a best fit procedure.
5. FRand is a Randomness Factor which describes the probability that dies in use one day are reused the day after, in relation with the just discussed die-box model. This factor proves to be of fundamental importance for the numerical data to fit well with the Crepusius data, as will be shown below.

Two extreme ways of operation of the die-box are possible:

- a. Every day the dies are placed back into the die-boxes (one for reverses, one for obverses) and the next day new pairs are randomly chosen,
- b. Once selected, the die is used until discarded. This means that it is affixed to its holder—a workstation (anvil) for obverses and a *signator*⁴⁷ sleeve for reverses.

Note that the hammer die at a workstation might have been interchanged with the one used the day before. FRand takes this into account.

Appendix 2. The detailed operation of MintSW

In what follows, the operation of *MintSW* is described in its different steps.

43 For more discussion, see Hersh, 1976.

44 Crawford, 1965.

45 See for example Sellwood, 1962, Rottinghaus, 2007, Faucher, 2009, Faucher, 2012, Faucher, 2016.

46 This example can be also estimated from the die counts of a given year. For example, the triumvirate of P.CREPVSI, M.LIMETAN and L.CENSOR (RRC 361-363) produced roughly 1500 dies (see below), by two engravers. This provides an estimate of $750/300\text{days} = 2.5$ dies/day. Of course, that implies that there were also two additional engravers for the reverse dies.

47 See Woytek, 2013.

Appendix 2.1 Creation of the die reservoirs, compliant with a modified lifetime statistic

The die lifetime can be measured by time or strikes;⁴⁸ at a constant minting rate, the two measures are proportional. However, in principle the best measure is strikes. Therefore, at the beginning of the computation, the MintSW die reservoirs (obverse and reverse) of each die are filled with a number of strikes (see Fig.3, top), obeying the exponential breakage law shown in Fig. A2.1. This number does not correspond to the real number, which is unknown, but to much lower values. However, these lower values are able to represent well the real mint procedure. This is done to reduce the computational time, because of the need to repeat the coinage several times to incorporate variance values of the stochastic process. These ‘strike containers’ for each die must not be confused with the die-boxes. Each die has a container of strikes, and the containers are placed in the die box. These containers are filled by using an exponential law of the die break probability as a function of time t:⁴⁹

$$Prob = \exp(-t/DLT),$$

to generate random probabilities between 0 and 1.

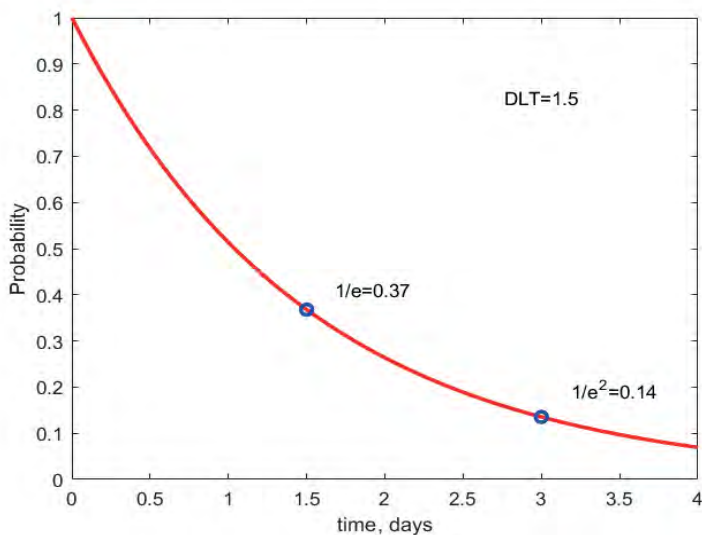


Fig.A2.1; Example of die life distribution, showing the probability at t=DLT and t=2 x DLT.

For each die, the corresponding die lifetime $t = - DLT \times \log(Prob)$ is computed and a related number of SW strikes (NSWS) assigned to that die. For example, if the random

48 See Mørkholm, 1983 for investigations on Greek dies, which are dated with a monthly precision and Albarède et al., 2021 for lifetime models which are, however, tailored mostly to Greek coinages. Greek coinages differ greatly from Roman ones, as demonstrated by the highly different lifetimes between obverse and reverse dies. In Roman coinage they are nearly equal, while in Greek coinage obverse dies last on average three times longer.

49 Esty, 2010.

probability is 0.37, then the Die Life Probability Distribution function in Fig.A2.1 calculates $t=1.5$ days. $DLT=1.5$ days, so $t=DLT$. Or for the probability of 0.14, Fig.A2.1 calculates $t=3$ days (which = $2 \times DLT$).

NSWS is proportional to the real and much higher number of strikes supported by that die, but must be selected so as to be able to describe with sufficient precision the break dynamics of the dies, as discussed above. The observed die pairs are in fact the only information that can be gained from the actual numismatic evidence.

By defining: $NSWS_1 = NSWS$ corresponding to $t = DLT$,

one has the explicit correspondence: $NSWS = NSWS_1 \times t / DLT$.

Therefore, in the two examples above, one gets $NSWS_1$ and $2 \times NSWS_1$ respectively. The time in MintSW is measured in days because this is the natural timescale of exchanges in the die-box model. Switching to hours would introduce another uncertainty, because we cannot be sure how many hours per day the mint operated. Since in MintSW all can be computed in days, we use that timescale. It is useful to note an interesting property of the adopted probability distribution. Integrating this distribution over t from 0 to infinity yields DLT as average die lifetime. The average number of coins per die corresponds to $NSWS_1$, which numismatists call the Characteristic Index.⁵⁰ The actual Crepusius sample has 414 obverse dies, 453 reverse dies and 3820 coins, so the Characteristic Index is around 9. $NSWS_1$ is set higher than 9, in order to produce a SW coinage larger than the real one. Since a too large $NSWS_1$ would increase the simulation time too much, values around 20 are used in MintSW, which guarantees approximately 12000 coins - a reasonable compromise.

The exponential probability distribution is a simplification, very useful to arrive at the simple Esty formula.⁵¹ In a more realistic model, possible within a numerical simulation like MintSW, one can limit the upper time boundary. Of course, an infinite lifetime is not possible; in reality, dies last until they break or are discarded due to wear. This latter case is not included in the exponential model. We know from modern numismatic experiments that for tetradrachms the dies become unusable at around 10,000 strikes.⁵² At that point, the original design degrades to a degree which is never observed in Crepusius denarii. The mintmasters took care to deliver a quality product.⁵³ Even if

50 Characteristic Index = Sample Size / Number of dies.

51 Esty, 2011.

52 Sellwood. 1963, Faucher, 2009, Faucher, 2016. Unfortunately, and strangely, no similar experiments are available for denarii. It is problematic to transfer tetradrachm results to denarii because tetradrachms have 3-4 times higher mass, which produces different hammer forces.

53 In sending his corpus of 3515 examples to Richard Schaefer on 1 Jan 2010, Ted Buttrey noted that 'Crepusius denarii rarely have serious die breaks', from which he inferred that the mint did not try to stretch die life, but immediately removed any that cracked. This implies the mint operated in high quality fashion. Private communication with Schaefer, March 2019.

not broken, over-used dies no longer guarantee a clear strike. This is the reason behind the introduction of the parameter DLmax (maximum Die Lifetime). This MintSW parameter has a visible impact in the Die Frequency Plots (see Fig.10-11).

Appendix 2.2 Diebox refilling scheme.

From the randomly created die reservoirs, MintSW's dieboxes are filled and maintained in proper operation. This means new dies are always available to replace discarded ones. The way dies are moved from die reservoirs to dieboxes is an important feature of MintSW. New dies refill the diebox when a minimum value is reached. At that point, BoxAdd dies ($\text{BoxAdd} = \text{FAdd} \times W$) are added to the diebox. These new dies are taken from the die reservoir in the same order they were placed in it, which simulates the actual engraving which was sequential.

Another possible choice would be to add the dies at regular time intervals, say once every *nundinum*.⁵⁴ However, it is inconceivable that production would have stopped until the engraver could cut more dies. Therefore a more flexible scheme must have been in operation, with engravers and the mint interacting. *MintSW*'s scheme is at least compliant with the need to guarantee continuous striking. The refill rate also must be compatible with the die engraving rate, which is assumed to be 2-3 dies per day at maximum, as previously pointed out. Thus *MintSW* controls both the diebox refill rate and the die engraving rate.

Appendix 2.3 SW sample

SW mintage starts with $\text{FAdd} \times W$ dies in the dieboxes. When the strikes assigned to a die (see A2.1 above) are used up, a new die is randomly taken from the diebox. At that time, a check is performed on the number of remaining dies in the diebox; if below the minimum allowed, $\text{FAdd} \times W$ dies are moved from the reservoir to the diebox.

At the beginning of the next working day, the dies are chosen according to FRand (see Section II, 5 above). Day after day, the above operations are repeated until all the strikes of one of the two reservoirs are finished. At that point the full SW sample is completed.

Appendix 2.4 Example of MintSW production

To show how the software works, Table A2 presents an example with 2 workstations and $\text{FAdd}=4$. The die boxes are filled with 8 dies ($\text{FAdd} \times W = 4 \times 2$), and random numbers of strikes are assigned to each die (columns 10-17, lines 2-3). The obverse numbers of strikes are in red; the reverse numbers in blue. Column 18 totals the number of strikes available.

⁵⁴ Period of nine days, ending with a market day.

Production starts on Day 1, by randomly selecting O and R dies, as exemplified. The day ends after 20 coins are produced. Note column 18 (lines 9 and 10) tracks the diminishing number of available strikes (and dies in some cases, when the die strikes are less than 20). All the data in terms of die pairs are recorded.

Production continues until the number of dies goes below Fadd, which occurs at the end of Day 2. At Day 3, 8 dies are added in the order they were engraved, in this case from 9 to 16. Production continues until all the die strikes are used, either for obverses or reverses.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
2		Anvil 1			Anvil 2			Die #	1	2	3	4	5	6	7	8	
3		Die pairs	Coins		Die pairs	Coins		O strikes	3	35	20	50	8	5	13	1	135
4								R strikes	27	9	2	38	14	43	25	32	190
5		O3-R7	20		O8-R2	1											Total Strikes
6					O4-R2	8											
7					O4-R1	11											
8	Day 1		20			20											
9								O strikes	3	35		31	8	5	13	•	95
10		O5-R1	8		O7-R5	13		R strikes	16		2	38	14	43	5	32	150
11		O2-R1	8		O1-R5	1											
12		O2-R4	4		O1-R8	2											
13					O4-R8	4											
14	Day 2		20			20											
15								O strikes		23		27		5			55
16								R strikes			2	34		43	5	26	110
17																	
18	Day 3	Refill the die boxes by 4 x 2 dies and continue															

Table A2: Example of minting with two workstations and FAdd = 4.

Appendix 2.5 Extraction of a reduced sample

This is an important step in *MintSW*. To make possible the comparison with a real coinage like *Crepusius*, the SW coinage must be reduced to the same size. This is performed by randomly extracting coins from the SW coinage until its size equals the real sample. This is called the *reduced* SW sample.

Appendix 2.6 Production of statistically significant data

The example in Table A2 is just one of infinitely many possible. Also, the actual numismatic sample is the result of similar random occurrences - the breaks of the dies, the usage of the coins, and our discovery of surviving coins. Thus, a comparison of the real sample with just one reduced SW sample would not be statistically meaningful. We aim to arrive at confidence intervals of the investigated parameters. To that end, the SW mintage must be applied multiple times (NMintages), exactly in the same way one tests the 50% probability of a flipped coin landing on its obverse. If one flipped a coin only 10

times, one could arrive at a wrong conclusion. Flipping many times tends to eliminate statistical fluctuations. We varied NMintages from 10 to 1000 and found consistent results starting from NMintages=20. Therefore, values in the range 20-100 are used. This increases the computation time but provides more reliable results. NMintages also is a fundamental control parameter needed to derive a confidence interval. In fact, for all the values determined, a mean value and a standard deviation are calculated.⁵⁵

Appendix 3. Example of *MintSW* applied to Fabius Pictor

Parameter	Abbr.	Units	Value
Engraved Obverse Dies	D _O	-	22
Engraved Reverse Dies	D _R	-	15
Workstations	W	-	1
Die Lifetime Obverse	DLT _O	Days	1.50
Die Lifetime Reverse	DLT _R	Days	1.15
Maximum Die Usage	DLmax	Days	3
Die Addition Factor	FAdd	-	4
Randomness Factor	FRand	%	20%

Subsidiary settings

Dies added to diebox	BoxAdd	-	Variable
Number of SW mintages	NMintages		Variable
Strike per day	NSWS_1		20

Fabius Pictor (*RRC* 268/1) is the simplest series usable by *MintSW* and thus can serve as a good guide. Fig. A3.1 shows the currently known dies and specimens, compared to the known dies in Michael Crawford's paper. The series uses the letters of the Latin alphabet as control marks for both the obverse and reverse dies. Fig.A3.1 arranges the reverse control letters across the top, and the obverse control letters along the vertical axis, missing letters included. Notice the imbalance between obverse and reverse dies, which makes the table rectangular (squared section marked in yellow). The whole alphabet is used for the reverses, plus an upside-down A (recently discovered). Clearly, the T reverse and L obverse dies broke early, while the missing P and Q obverses might be explained in two ways. In the simpler one, P and Q broke very early. Alternatively, the R die was chosen earlier from the diebox than P and Q and then production ended, so P and Q remained unused.

⁵⁵ For more details and discussion, refer to e.g. Fig. 8 and 9 and related discussions.



Fig. A3.1: Matrix of the die pairs of RRC 268/1b. The reverse dies are listed across the top; the obverse dies are listed along the vertical axis. The blue fields denote the pairs known in Crawford, 1966, the orange those discovered by Schaefer. The intersection numbers give the known specimens from Schaefer’s data (May 2022 update). In summary:

	<i>Sample</i>	<i>O dies</i>	<i>R dies</i>	<i>O Sing.</i>	<i>R Sing.</i>	<i>Pairs</i>	<i>Pair Sing.</i>
Crawford:	105	14	18	0	0	33	2
Schaefer:	379	15	22	0	2	46	7

Table A3: Table of *MintSW* parameters, with the values used for the simulation of RRC 268/1b.

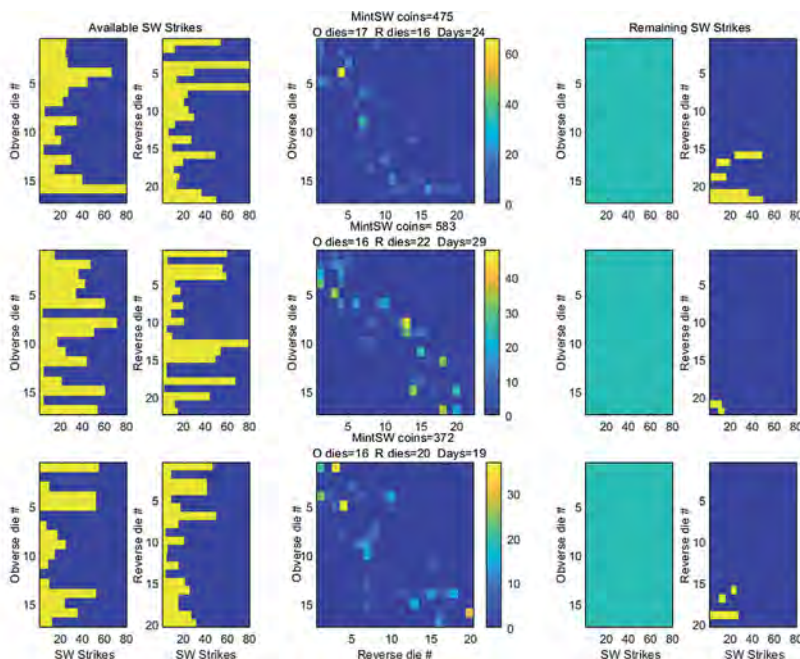


Fig. A3.2: Three different possible SW mintages of RRC 268/1 (one per line) showing both the randomness of the dies and of their pairings. On each line, the two plots at the left show the die reservoirs, filled assuming NSW_S_1=20. The centre plot presents the mintage matrix. Since this plot is too small to insert numbers in each die pair box, each box is coloured and the vertical bar at right gives the number for every colour. Deep blue means zero. The two rightmost plots show the status of the die reservoirs after the minting. ‘SW Strikes’ = the number of software coins struck and ‘Days’ = the days needed to finish the coinage. Parameters as in Table 1.

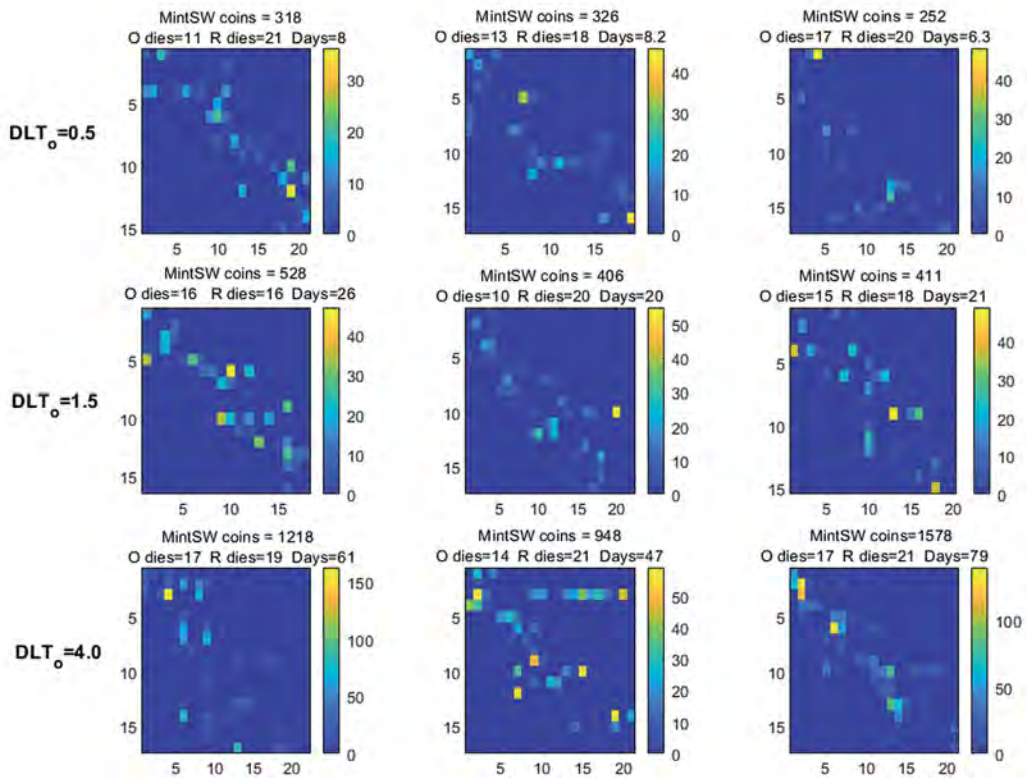


Fig. A3.3: Effect of varying DLT_0 in *MintSW* simulations of RRC 268/1b. All plots have same format and titles as the centre plots in Fig. A3.2. In line 1, $DLT_0=0.5$; in line 2, $DLT_0=1.5$; in line 3, $DLT_0=4$. The 3 plots in each line represent different productions, but with the same nominal parameters (see Table A3). All vertical axes designate the Obv. die number; all horizontal axes the Rev. die number, as in Fig. A3.2.

This coinage is useful to illustrate how *MintSW* produces sets of obverse and reverse dies, and then couples them into a matrix of die pair strikes similar to Fig. A3.1. The matrixes of 3 *MintSW* productions are shown in Fig. A3.2, along with the die reservoirs before and after production. On the horizontal axis of the reservoirs the strikes of each die are shown. This makes clear the randomness of die longevity and how that may strongly impact the die pairings. The number of *MintSW* coins (SW Strikes) and the corresponding mintage time well demonstrate the strong variations of the exponential lifetime model. To gain some feeling of *MintSW* parameter effects on the mintage patterns, the results of varying DLT_0 are shown in Fig. A3.3.

In Fig. A3.3 DLT_0 is varied using the values proposed by Buttrey, Witschonke and an intermediate one. The plots show that varying DLT_0 does not cause significant differences in the mintage patterns or die numbers. However, it does strongly influence the mintage times, which increase proportionally to DLT_0 . We can see this by starting with any plot in the top row, and then looking at the plots directly below.

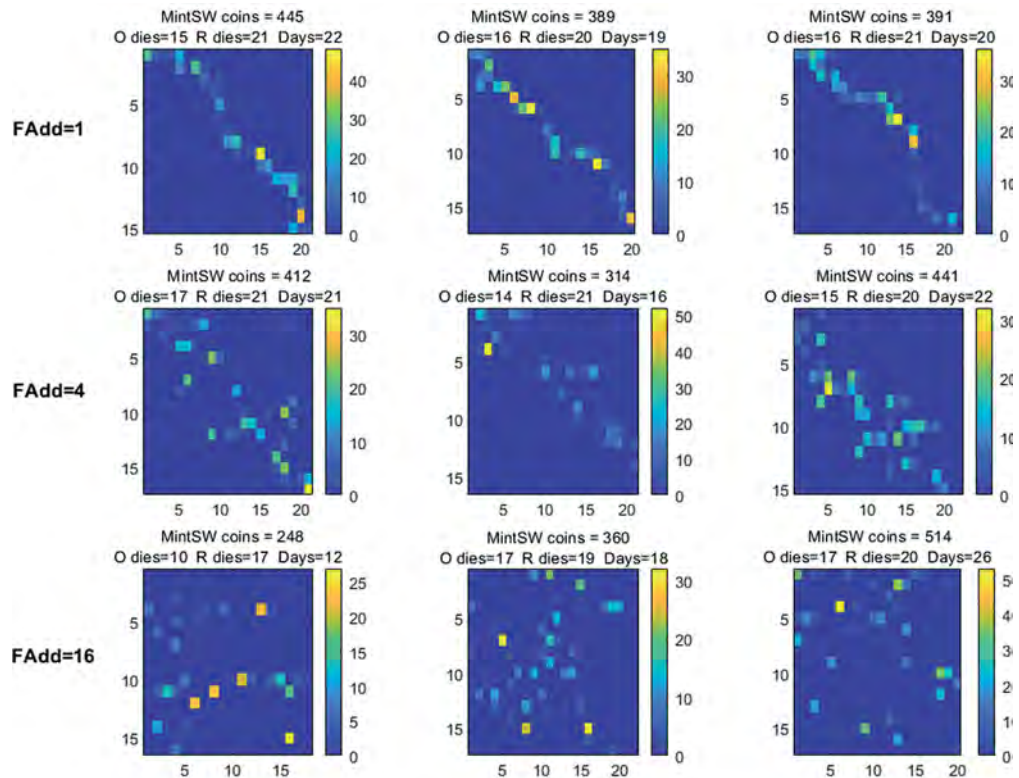


Fig. A3.4: Effect of varying FAdd in *MintSW* simulations of RRC 268/1b. All plots have same format and titles as the center plots in Fig.A3.2. In line 1, FAdd=1; in line 2, FAdd=4; in line 3, FAdd=16. The 3 plots in each line represent different productions, but with the same nominal parameters (see Table A3). All vertical axes designate the Obv. die number; all horizontal axes the Rev. die number.

In Fig. A3.4 the effects of FAdd, which governs the way the diebox is managed, are presented. A too small value (FAdd=1) does not permit many pairs to appear and creates mainly linear patterns (see the top line of plots). A too large value (FAdd=16), meaning that almost all the dies of the coinage are in the diebox from the beginning, makes pairs appear almost everywhere in the matrix (see the bottom line of plots). Values of FAdd from 3 to 6 result in patterns (see middle line of plots where FAdd=4) similar to the actual Fabius Pictor material.

Appendix 4. The *Crepusius* data

Here is the *Crepusius* data updated to Sept. 2021, the result of three generations of researchers. As discussed earlier, the data were entered in an Excel matrix (see Fig.1), which was then transformed to Table A4.2 shown below. The table presents a catalogue sorted by obverse dies at the left, and reverse dies at the right. The columns of the obverse catalogue denote:

1. Progressive numbering of the dies,
2. Progressive numbering of the symbols, according to Hersh order,
3. Obverse symbol — for 1(Nil) a blank is behind Apollo; symbols 2-25 are below Apollo's chin,
4. Obverse Letter — for 1(Nil) the letter is below Apollo's chin; for symbols 2-25 the letter is behind Apollo's head,
5. Number of die pairs known,
6. et seq.) The reverse dies (identified by reverse number) paired with this obverse die, followed by the number of coins known. For example, Obv.35 (Fulmen and N), has been found on 3 die pairs—on 11 coins paired with reverse die 50; on 13 coins paired with reverse die 65; and on 1 coin paired with reverse die 77. A red number means the reverse die is a duplicate (see the reverse catalogue below).

The columns of the reverse catalogue denote:

1. Reverse number in Arabic numerals (Roman numerals are on the coins). Those in yellow are duplicated dies. For example, 4.5 is one of two dies known with Roman numeral 4,
2. Number of pairs known,
3. et seq.) The obverse dies paired with this reverse die, each obverse followed by the number of coins known. For example, Rev.15 has been found on 3 die pairs—on 11 coins paired with obverse die 1-A; on one coin paired with obverse die 1-B; and on 7 coins paired with obverse die 3-D. Obverse die 1-A means the die with symbol Nil and letter A; obverse 3-D means the die with symbol Feather and letter D.

19 coins, occurring in 9 die pairs, have unreadable reverse numerals. Their obverses, followed by the number of examples known, are:

4-M 2 4-P 2 6-N 3 8-B 3 15-R 1 17-B 1 17-K 1 18-S 1 19-S 5

Table A4.1 shows the die and die pair frequency distributions, and in Fig. A4.1 they are plotted. Note that the randomness of the obverse and reverse die lifetimes causes a greater frequency decay of the die pairs. Obverse and reverse die frequencies are populated by more than one die until about 28, but the die pairs only until 20.

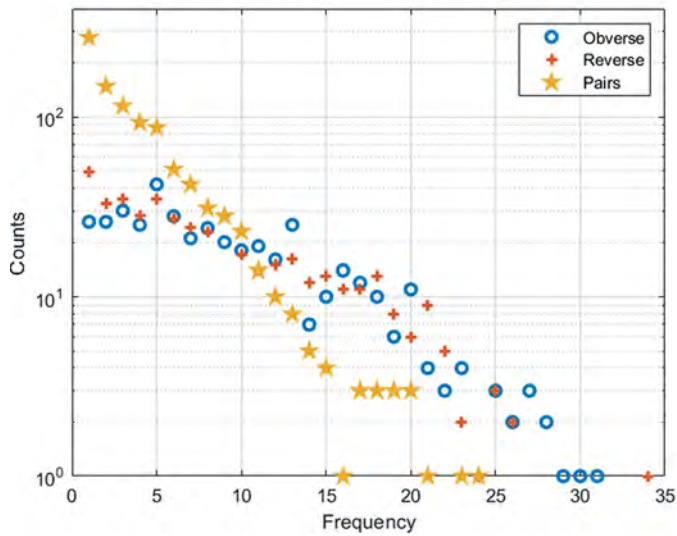


Fig.A4.1 Frequency plot with Counts on a logarithmic scale.

Frequency	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	TOT		
Obverse	26	26	30	25	42	28	21	24	20	18	19	16	25	7	10	14	12	10	6	11	4	3	4	0	3	2	3	2	1	1	1	0	0	0	0	0	0	0	0	0	414		
Reverse	49	33	35	28	35	27	24	23	28	17	14	15	16	12	13	11	11	13	8	6	9	5	2	1	3	2	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	442
Pairs	277	148	115	93	87	51	42	31	28	23	14	10	8	5	4	1	3	3	3	3	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	952	

Table A4.1: Crepusius frequency table for obverse dies, reverse dies and die pairs.

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